Technical Means of Automation

RC Circuits as Noise Filters

Institute of Information Engineering, Automation and Mathematics

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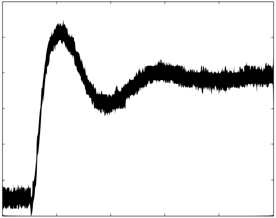
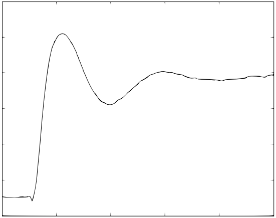
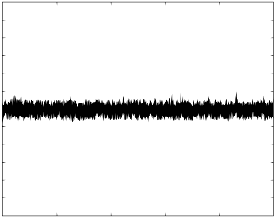
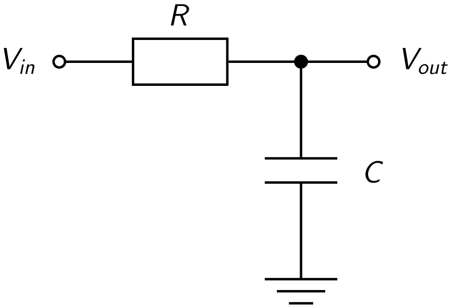
Noise

measured signal = useful signal + noise

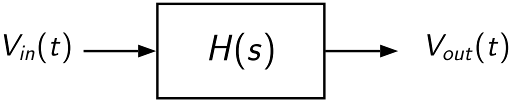




RC Low-Pass Filter



$$V_{\text{in}} = f(t), \quad V_{\text{out}} = f(t)$$

RC Circuit as Transfer Function



$$H(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)}$$

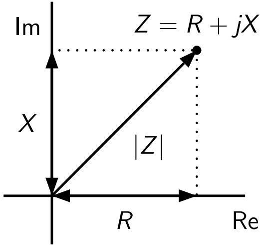
Electrical Impedance

Impedance (Z in Ω) is a comprehensive expression of any and all forms of **opposition to electron flow**, including both **resistance** and **reactance**. It is a **complex** variable.

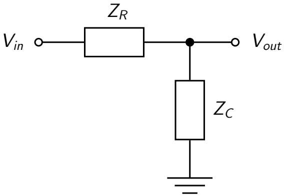
$$Z = R + jX$$

R - **resistance** to direct current

X - **reactance** - resistance to alternating current



Impedance of Circuit



Transient equation of circuit (same as for voltage divider).

$$V_{\text{out}} = \frac{Z_C}{Z_R + Z_C} V_{\text{in}}$$

Impedance of resistor is just resistance (reactance is not present).

$$Z_R = R + jX_R = R, \quad X_R = 0$$

Impedance of capacitor is just reactance (resistance is not present).

$$Z_C = R_C + jX_C = jX_C, \quad R_C = 0$$

Impedance of Capacitor

Ohm's law for AC circuits (quantities vary in time).

$$Z_C(t) = \frac{V_C(t)}{I_C(t)}$$

Voltage across the capacitor has a harmonic waveform in AC circuit.

$$V_C(t) = A\sin(\omega t)$$

Current through capacitor.

$$I_C(t) = C\frac{dV_C(t)}{dt}$$

$$I_C(t) = C\frac{d(A\sin(\omega t))}{dt} = CA\omega\cos(\omega t)$$

Impedance of Capacitor

Impedance of capacitor from Ohm's law.

$$Z_C(t) = \frac{A\sin(\omega t)}{CA\omega\cos(\omega t)}$$

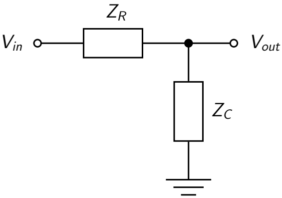
$$Z_C(t) = \frac{\sin(\omega t)}{C\omega\sin\left(\omega t + \frac{\pi}{2}\right)}$$

$$Z_C(s) = \mathcal{L}\{Z_C(t)\} = \frac{\frac{\omega}{s^2+\omega^2}}{C\omega\frac{s\sin\left(\frac{\pi}{2}\right)+\omega\cos\left(\frac{\pi}{2}\right)}{s^2+\omega^2}}$$

$$Z_C(s) = \frac{\omega}{C\omega(s\times 1+\omega\times 0)}$$

$$Z_C(s) = \frac{1}{Cs}$$

Transfer Function of RC Circuit



In time domain.

$$V_{\text{out}}(t) = \frac{Z_C(t)}{Z_R + Z_C(t)} V_{\text{in}}(t)$$

Laplace transform $\rightarrow$ Transfer function.

$$H(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{Z_C(s)}{Z_R + Z_C(s)}$$

Transfer Function of RC Circuit

$$H(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{Z_C(s)}{Z_R + Z_C(s)}$$

$$H(s) = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{RCs + 1}$$

Frequency domain transfer function.

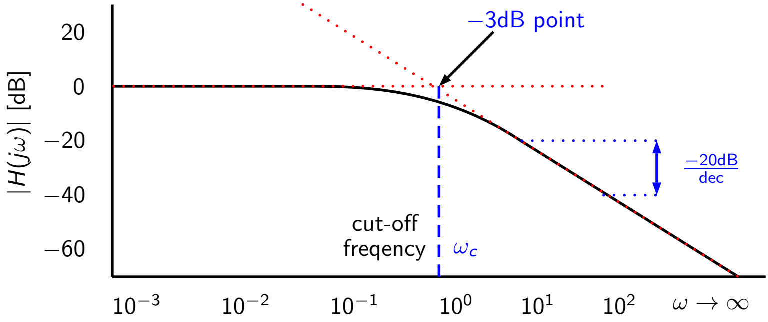
$$s = j\omega$$

$$H(j\omega) = \frac{V_{\text{out}}(j\omega)}{V_{\text{in}}(j\omega)} = \frac{1}{RCj\omega + 1}$$

Amplitude vs. Frequency

Amplitude as a function of angular frequency:

$$|H(j\omega)| = \frac{|V_{\text{out}}(j\omega)|}{|V_{\text{in}}(j\omega)|} = \frac{1}{\sqrt{1 + R^2C^2\omega^2}}$$



Amplitude vs. Frequency

Cut-off frequency is the point where the amplitude starts to decrease rapidly with the drop-off of -20dB per decade of frequency rise.
ω_c - cut-off frequency (a.k.a. corner frequency).
Is also known as half power point, where:

$$|H(j\omega_c)|^2 = \frac{1}{2} \quad \text{or} \quad |H(j\omega_c)| = \frac{1}{\sqrt{2}}$$

$$|H(j\omega_c)|^2 = \frac{1}{2} = \frac{1}{1 + R^2C^2{\omega_c}^2}$$

$$\omega_c = \frac{1}{RC} \quad \text{or} \quad f_c = \frac{1}{2\pi RC}$$

Signal Gain and Loss

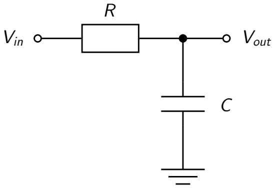
The **decibel** is an auxiliary unit that indicates the **ratio** of two field quantities (voltage) or energy quantities (power).

Voltage gain: $$G_V = 20 \times \log_{10}\left(\frac{|V_{\text{out}}|}{|V_{\text{in}}|}\right) = 10 \times \log_{10}\left(\frac{|V_{\text{out}}|}{|V_{\text{in}}|}\right)^2 \quad \textbf{in dB}$$

Power gain: $$G_p = 10 \times \log_{10}\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) \quad \textbf{in dB}, \quad P \approx V^2$$

Task

Consider the RC low-pass filter on the figure. The input voltage $V_{\text{in}}(t)$ contains useful signal (sensor reading) and noise with much higher dominant frequency $f_n = 2.5kHz$. The RC circuit contains capacitor with $C = 1\mu\text{F}$.



1. Calculate the resistor value R, to suppress the amplitude of output voltage $V_{\text{out}}(t)$ to 10% of original amplitude of $V_{\text{in}}(t)$.
2. Calculate the value of cut-off frequency in Hz.