I Tabuľky neurčitých integrálov

I.1 Neurčité integrály elementárnych a niektorých iných funkcií

$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C,\ a \in \mathbb{R} - \{-1\},\ x > 0$

$\int \frac{1}{x} \,\mathrm{d}x = \ln|x| + C$

$\int \sin x \,\mathrm{d}x = -\cos x + C$

$\int \cos x \,\mathrm{d}x = \sin x + C$

$\int \frac{1}{\cos^2 x} \,\mathrm{d}x = \mathrm{tg}\, x + C$

$\int \frac{1}{\sin^2 x} \,\mathrm{d}x = -\mathrm{cotg}\, x + C$

$\int \mathrm{e}^x \,\mathrm{d}x = \mathrm{e}^x + C$

$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C, \quad a > 0,\ a \neq 1$

$\int \frac{1}{1+x^2} \,\mathrm{d}x = \mathrm{arctg}\, x + C$

$\int \frac{1}{\sqrt{1-x^2}} \,\mathrm{d}x = \arcsin x + C$

$\int \frac{1}{a^2+x^2} \,\mathrm{d}x = \frac{1}{a} \mathrm{arctg}\, \frac{x}{a} + C, \quad a > 0$

$\int \frac{1}{\sqrt{a^2-x^2}} \,\mathrm{d}x = \arcsin \frac{x}{a} + C, \quad a > 0$

$\int \frac{1}{a^2-x^2} \,\mathrm{d}x = \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| + C, \quad a > 0$

$\int \frac{1}{ax^2+bx+c} \,\mathrm{d}x = \frac{2}{\sqrt{4ac-b^2}} \mathrm{arctg}\, \frac{2ax+b}{\sqrt{4ac-b^2}}, \quad b^2 < 4ac$

$\int \frac{1}{ax^2+bx+c} \,\mathrm{d}x = \frac{1}{\sqrt{b^2-4ac}} \ln\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right|, \quad b^2 > 4ac$

$\int \sqrt{x^2+px+q} \,\mathrm{d}x = \frac{2x+p}{4} \sqrt{x^2+px+q} + \frac{4q-p^2}{8} \ln\left|2\sqrt{x^2+px+q} + 2x + p\right|$

$\int \sqrt{-x^2+px+q} \,\mathrm{d}x = \frac{-2x+p}{-4} \sqrt{-x^2+px+q} + \frac{4q+p^2}{-8} \arcsin \frac{-2x+p}{\sqrt{p^2+4q}}$

$\int \frac{1}{\sqrt{x^2+px+q}} \,\mathrm{d}x = \ln\left|2x+p+2\sqrt{x^2+px+q}\right|$

$\int \frac{1}{\sqrt{-x^2+px+q}} \,\mathrm{d}x = -\arcsin \frac{-2x+p}{\sqrt{p^2+4q}}$

$\int \sin px \sin qx \,\mathrm{d}x = \frac{\sin(q-p)x}{2(q-p)} - \frac{\sin(q+p)x}{2(q+p)}$

$\int \cos px \cos qx \,\mathrm{d}x = \frac{\sin(p+q)x}{2(p+q)} + \frac{\sin(p-q)x}{2(p-q)}$

$\int \sin px \cos qx \,\mathrm{d}x = -\frac{\cos(p-q)x}{2(p-q)} - \frac{\cos(q+p)x}{2(q+p)}$

I.2 Neurčité integrály racionálnych funkcií

$$I_{n,m} = \int \frac{x^n}{(a+bx)^m} \,\mathrm{d}x, \qquad n = 0,1,2\ldots, \quad m = 1,2,3,\ldots, \quad a,b \in \mathbb{R} - \{0\}$$

$I_{0,1} = \frac{1}{b} \ln|a+bx|$

$I_{0,m} = \frac{1}{(1-m)b(a+bx)^{m-1}}, \quad m \geq 2$

$I_{1,1} = \frac{1}{b}\left(x - \frac{a}{b}\ln|a+bx|\right)$

$I_{1,2} = \frac{1}{b^2}\left(\frac{a}{a+bx} + \ln|a+bx|\right)$

$I_{1,m} = \frac{1}{b^2}\left(\frac{1}{(2-m)(a+bx)^{m-2}} + \frac{a}{(m-1)(a+bx)^{m-1}}\right), \quad m \geq 3$

$I_{2,1} = \frac{1}{b}\left(\frac{x^2}{2} - \frac{a}{b}x + \frac{a^2}{b^2}\ln|a+bx|\right)$

$I_{2,2} = \frac{1}{b^2}\left[x - \frac{a}{b}\left(\frac{a}{a+bx} + 2\right)\right]$

$I_{2,3} = \frac{1}{b^3}\left(\frac{2a}{a+bx} - \frac{a^2}{2(a+bx)^2} + \ln|a+bx|\right)$

$I_{2,m} = \frac{1}{b^3}\left(\frac{1}{(3-m)(a+bx)^{m-3}} + \frac{2a}{(m-2)(a+bx)^{m-2}} - \frac{a^2}{(m-1)(a+bx)^{m-1}}\right), \quad m \geq 4$

$I_{n,1} = \sum_{i=0}^{n-1} \frac{(-1)^i a^i x^{n-i}}{(n-i)b^{i+1}} + \frac{(-a)^n}{b^{n+1}} \ln|a+bx|, \quad n \geq 1$

$I_{n,m} = \frac{x^n}{(1-m)b(a+bx)^{m-1}} + \frac{n}{(m-1)b} I_{n-1,m-1}, \quad m \geq 2$

$$I_{n,m} = \int \frac{1}{x^n(a+bx)^m}\,\mathrm{d}x, \qquad n = 1,2,3\ldots, \quad m = 1,2,3,\ldots, \quad a,b \in \mathbb{R}-\{0\}$$

$I_{1,1} = -\frac{1}{a}\ln|\frac{a}{x}+b|$

$I_{1,2} = \frac{1}{a}\left(\frac{1}{a+bx} - \frac{1}{a}\ln|\frac{a}{x}+b|\right)$

$I_{1,m} = \sum_{i=1}^{m-1}\frac{1}{ia^{m-i}(a+bx)^i} - \frac{1}{a^m}\ln|\frac{a}{x}+b|, \quad m \geq 2$

$I_{2,1} = -\frac{1}{a}\left(\frac{1}{x} - \frac{b}{a}\ln|\frac{a}{x}+b|\right)$

$I_{2,2} = -\frac{1}{a^2}\left(\frac{b}{a+bx} + \frac{1}{x} - \frac{2b}{a}\ln|\frac{a}{x}+b|\right)$

$I_{2,m} = \frac{-1}{ax(a+bx)^{m-1}} - \frac{mb}{a}I_{1,m}$

$I_{3,2} = \frac{1}{a^2}\left(\frac{b^2}{a(a+bx)} + \frac{2b}{ax} - \frac{1}{2x^2} - \frac{3b^2}{a^2}\ln|\frac{a}{x}+b|\right)$

$I_{3,3} = \frac{1}{a^3}\left(\frac{3b^2}{a(a+bx)} + \frac{b^2}{2(a+bx)^2} + \frac{3b}{ax} - \frac{1}{2x^2} - \frac{6b^2}{a^2}\ln|\frac{a}{x}+b|\right)$

$I_{3,m} = \frac{(m+1)bx-a}{2a^2x^2(a+bx)^{m-1}} + \frac{m(m+1)b^2}{2a^2}I_{1,m}, \quad m \geq 4$

$I_{n,1} = \sum_{i=1}^{n-1}\frac{(-1)^i b^{i-1}}{(n-i)a^i x^{n-i}} + \frac{(-1)^{n-1}b^{n-1}}{a^n}\ln|\frac{x}{a+bx}|, \quad n \geq 2$

$$I_{n,m} = \int \frac{x^n}{(a^2+b^2x^2)^m}\,\mathrm{d}x, \qquad n = 0,1,2\ldots, \quad m = 1,2,3,\ldots, \quad a>0,\ b>0$$

$I_{0,1} = \frac{1}{ab}\ \mathrm{arctg}\,\frac{bx}{a}$

$I_{0,2} = \frac{x}{2a^2(a^2+b^2x^2)} + \frac{1}{2a^3b}\ \mathrm{arctg}\,\frac{bx}{a}$

$I_{0,m} = \frac{x}{2(m-1)a^2(a^2+b^2x^2)^{m-1}} + \frac{2m-3}{2(m-1)a^2}I_{0,m-1}, \quad m \geq 2$

$I_{1,1} = \frac{1}{2b^2}\ln(a^2+b^2x^2)$

$I_{1,2} = -\frac{1}{2b^2(a^2+b^2x^2)}$

$I_{1,m} = -\frac{1}{2(m-1)b^2(a^2+b^2x^2)^{m-1}}, \quad m \geq 2$

$I_{2,1} = \frac{x}{b^2} - \frac{a}{b^3}\ \mathrm{arctg}\,\frac{bx}{a}$

$I_{2,2} = \frac{-x}{2b^2(a^2+b^2x^2)} + \frac{1}{2ab^3}\ \mathrm{arctg}\,\frac{bx}{a}$

$I_{2,m} = \frac{-x}{2(m-1)b^2(a^2+b^2x^2)^{m-1}} + \frac{1}{2(m-1)b^2}I_{0,m-1}, \quad m \geq 2$

$I_{n,m} = \frac{-x^{n-1}}{2(m-1)b^2(a^2+b^2x^2)^{m-1}} + \frac{n-1}{2(m-1)b^2}I_{n-2,m-1}, \quad m \geq 2,\ n \geq 2$

$$I_{n,m} = \int \frac{1}{x^n(a^2+b^2x^2)^m}\,\mathrm{d}x, \qquad n = 1,2,3,\ldots, \quad m = 1,2,3,\ldots, \quad a>0,\ b>0$$

$I_{1,1} = \frac{1}{2a^2}\ln\frac{x^2}{a^2+b^2x^2}$

$I_{1,2} = \frac{1}{2a^2(a^2+b^2x^2)} + \frac{1}{2a^4}\ln\frac{x^2}{a^2+b^2x^2}$

$I_{1,m} = \frac{1}{2(m-1)a^2(a^2+b^2x^2)^{m-1}} + \frac{1}{a^2}I_{1,m-1}, \quad m \geq 2$

$I_{2,1} = \frac{-1}{a^2x} + \frac{b}{a^3}\ \mathrm{arctg}\ \frac{bx}{a}$

$I_{2,2} = \frac{1}{a^4x} - \frac{b^2x}{2a^4(a^2+b^2x^2)} - \frac{3b}{2a^5}\ \mathrm{arctg}\ \frac{bx}{a}$

$I_{2,m} = -\frac{1}{a^2x(a^2+b^2x^2)^{m-1}} - \frac{2(m-1)b^2}{a^2} I_{0,m}, \quad m \geq 2$

$I_{3,1} = -\frac{1}{2a^2x^2} - \frac{b^2}{2a^4} \ln \frac{x^2}{a^2+b^2x^2}$

$I_{3,2} = -\frac{1}{2a^4x^2} - \frac{b^2}{2a^4(a^2+b^2x^2)} - \frac{b^2}{a^6} \ln \frac{x^2}{a^2+b^2x^2}$

$I_{n,m} = \frac{1}{(1-n)a^2x^{n-1}(a^2+b^2x^2)^{m-1}} - \frac{(2m+n-3)b^2}{(n-1)a^2} I_{n-2,m}, \quad n \geq 3, m \geq 3$

$$I_{n,m} = \int \frac{x^n}{(a^2-b^2x^2)^m}\,\mathrm{d}x, \qquad n = 0,1,2\ldots, \quad m = 1,2,3,\ldots, \quad a>0,\ b>0$$

$I_{0,1} = \frac{1}{2ab} \ln \left|\frac{a+bx}{a-bx}\right|$

$I_{0,2} = \frac{x}{2a^2(a^2-b^2x^2)} + \frac{1}{4a^3b} \ln \left|\frac{a+bx}{a-bx}\right|$

$I_{0,m} = \frac{x}{2(m-1)a^2(a^2-b^2x^2)^{m-1}} + \frac{2m-3}{2(m-1)a^2} I_{0,m-1}, \quad m \geq 2$

$I_{1,1} = -\frac{1}{2b^2} \ln |a^2-b^2x^2|$

$I_{1,2} = \frac{1}{2b^2(a^2-b^2x^2)}$

$I_{1,m} = \frac{1}{2(m-1)b^2(a^2-b^2x^2)^{m-1}}, \quad m \geq 2$

$I_{2,1} = -\frac{x}{b^2} + \frac{a}{2b^3} \ln \left|\frac{a+bx}{a-bx}\right|$

$I_{2,2} = \frac{x}{2b^2(a^2-b^2x^2)} - \frac{1}{4ab^3} \ln \left|\frac{a+bx}{a-bx}\right|$

$I_{2,m} = \frac{x}{2(m-1)b^2(a^2-b^2x^2)^{m-1}} - \frac{1}{2(m-1)b^2} I_{0,m-1}, \quad m \geq 2$

$I_{3,1} = -\frac{x^2}{2b^2} - \frac{a^2}{2b^4} \ln |a^2-b^2x^2|$

$I_{3,2} = \frac{a^2}{2b^4(a^2-b^2x^2)} + \frac{1}{2b^4} \ln |a^2-b^2x^2|$

$I_{n,m} = \frac{x^{n-1}}{2(m-1)b^2(a^2-b^2x^2)^{m-1}} - \frac{n-1}{2(m-1)b^2} I_{n-2,m-1}, \quad n \geq 2, m \geq 2$

$$I_{n,m} = \int \frac{1}{x^n(a^2-b^2x^2)^m}\,\mathrm{d}x, \qquad n = 1,2,3\ldots, \quad m = 1,2,3,\ldots, \quad a>0,\ b>0$$

$I_{1,1} = \frac{1}{2a^2} \ln \left|\frac{x^2}{a^2-b^2x^2}\right|$

$I_{1,2} = \frac{1}{2a^2(a^2-b^2x^2)} + \frac{1}{2a^4} \ln \left|\frac{x^2}{a^2-b^2x^2}\right|$

$I_{1,m} = \frac{1}{2(m-1)a^2(a^2-b^2x^2)^{m-1}} + \frac{1}{a^2} I_{1,m-1}, \quad m \geq 2$

$I_{2,1} = \frac{-1}{a^2x} + \frac{b}{2a^3} \ln \left|\frac{a+bx}{a-bx}\right|$

$I_{2,2} = \frac{-1}{a^4x} + \frac{b^2x}{2a^4(a^2-b^2x^2)} + \frac{3b}{4a^5} \ln \left|\frac{a+bx}{a-bx}\right|$

$I_{2,m} = \frac{-1}{a^2x(a^2-b^2x^2)^{m-1}} + \frac{b^2(2m-1)}{a^2} I_{0,m}, \quad m \geq 2$

$I_{3,1} = \frac{-1}{2a^2x^2} + \frac{b^2}{2a^4} \ln \left|\frac{x^2}{a^2-b^2x^2}\right|$

$I_{3,2} = \frac{-1}{2a^4x^2} + \frac{b^2}{2a^4(a^2-b^2x^2)} + \frac{b^2}{a^6} \ln \left|\frac{x^2}{a^2-b^2x^2}\right|$

$I_{n,m} = \frac{-1}{(n-1)a^2x^{n-1}(a^2-b^2x^2)^{m-1}} + \frac{(2m+n-3)b^2}{(n-1)a^2} I_{n-2,m}, \quad n \geq 2, m \geq 2$

I.3 Neurčité integrály iracionálnych funkcií

$$I_{n,m} = \int \frac{x^n}{\sqrt{(a+bx)^m}}\,\mathrm{d}x, \qquad n = 0,1,2,\dots, \quad m = 1,3,5,\dots, \quad ab \neq 0$$

$I_{0,1} = \frac{2}{b}\sqrt{a+bx}$

$I_{0,m} = \frac{2}{(2-m)b\sqrt{(a+bx)^{m-2}}}, \quad m \geq 2$

$I_{1,1} = \frac{2\sqrt{a+bx}}{3b^2}(bx-2a)$

$I_{1,m} = \frac{2}{b^2\sqrt{(a+bx)^{m-2}}}\left(\frac{a+bx}{4-m} + \frac{a}{m-2}\right), \quad m \geq 3$

$I_{2,1} = \frac{2\sqrt{a+bx}}{b^3}\left(\frac{(a+bx)^2}{5} - \frac{2a(a+bx)}{3} + a^2\right)$

$I_{2,m} = \frac{2}{b^3\sqrt{(a+bx)^{m-2}}}\left(\frac{(a+bx)^2}{6-m} + \frac{2(a+bx)}{m-4} - \frac{a^2}{m-2}\right), \quad m \geq 2$

$I_{3,1} = \frac{2\sqrt{a+bx}}{b^4}\left(\frac{(a+bx)^3}{7} - \frac{3(a+bx)^2.a}{5} + a^2(a+bx) - a^3\right)$

$I_{3,3} = \frac{2}{b^4\sqrt{a+bx}}\left(\frac{(a+bx)^3}{5} - a(a+bx)^2 + 3a^2(a+bx) + a^3\right)$

$I_{n,m} = \frac{2}{b^{n+1}\sqrt{(a+bx)^{m-2}}}\sum_{i=0}^{n}\frac{(-1)^i\binom{n}{i}(a+bx)^{n-i}a^i}{2n-2i-m+2}, \quad n \geq 3, m \geq 3$

$$I_{n,m} = \int \frac{1}{x^n\sqrt{(a+bx)^m}}\,\mathrm{d}x, \qquad n = 1,2,3\dots, \quad m = 1,3,5,\dots, \quad ab \neq 0$$

$I_{1,1} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}\right|, \quad \text{ak } a > 0$

$I_{1,1} = \frac{2}{\sqrt{-a}}\operatorname{arctg}\frac{\sqrt{a+bx}}{\sqrt{-a}}, \quad \text{ak } a < 0$

$I_{1,3} = \frac{2}{a\sqrt{a+bx}} + \frac{1}{a}I_{1,1}$

$I_{1,m} = \frac{2}{(m-2)a\sqrt{(a+bx)^{m-2}}} + \frac{1}{a}I_{1,m-2}, \quad m \geq 3$

$I_{2,1} = -\frac{\sqrt{a+bx}}{ax} - \frac{b}{2a}I_{1,1}$

$I_{2,3} = -\frac{a+3bx}{a^2x\sqrt{a+bx}} - \frac{3b}{2a^2}I_{1,1}$

$I_{2,m} = \frac{-1}{ax\sqrt{(a+bx)^{m-2}}} - \frac{mb}{2a}I_{1,m}, \quad m \geq 3$

$I_{3,1} = \frac{3bx-2a}{4a^2x^2}\sqrt{a+bx} + \frac{3b^2}{8a^2}I_{1,1}$

$I_{n,1} = -\frac{\sqrt{a+bx}}{(n-1)ax^{n-1}} - \frac{(2n-3)b}{2(n-1)a}I_{n-1,1}, \quad n \geq 2$

$I_{n,m} = \frac{2}{(2-m)bx^n\sqrt{(a+bx)^{m-2}}} - \frac{2n}{(m-2)b}I_{n+1,m-2}, \quad m \geq 3$

$$I_{n,m} = \int x^n\sqrt{(a+bx)^m}\,\mathrm{d}x, \qquad n = 0,1,2,\dots, \quad m = 1,3,5,\dots$$

$I_{0,1} = \frac{2\sqrt{(a+bx)^3}}{3b}$

$I_{0,m} = \frac{2\sqrt{(a+bx)^{m+2}}}{(m+2)b}$

$I_{1,1} = \frac{2}{b^2}\left(\frac{\sqrt{(a+bx)^5}}{5} - \frac{a\sqrt{(a+bx)^3}}{3}\right)$

$$I_{1,m} = \frac{2}{b^2}\left(\frac{\sqrt{(a+bx)^{m+4}}}{m+4} - \frac{a\sqrt{(a+bx)^{m+2}}}{m+2}\right)$$
$$I_{2,1} = \frac{2}{b^3}\left(\frac{\sqrt{(a+bx)^7}}{7} - \frac{2a\sqrt{(a+bx)^5}}{5} + \frac{a^2\sqrt{(a+bx)^3}}{3}\right)$$
$$I_{2,m} = \frac{2}{b^3}\left(\frac{\sqrt{(a+bx)^{m+6}}}{m+6} - \frac{2a\sqrt{(a+bx)^{m+4}}}{m+4} + \frac{a^2\sqrt{(a+bx)^{m+2}}}{m+2}\right)$$
$$I_{3,1} = \frac{2\sqrt{(a+bx)^3}}{b^4}\left(\frac{(a+bx)^3}{9} - \frac{3(a+bx)^2a}{7} + \frac{3(a+bx)a^2}{5} - \frac{a^3}{3}\right)$$
$$I_{3,m} = \frac{2\sqrt{(a+bx)^{m+2}}}{b^4}\left(\frac{(a+bx)^3}{8+m} - \frac{3(a+bx)^2a}{6+m} + \frac{3(a+bx)a^2}{4+m} - \frac{a^3}{2+m}\right)$$
$$I_{n,m} = \frac{2\sqrt{(a+bx)^{m+2}}}{b^{n+1}}\sum_{i=0}^{n}\frac{(-1)^i\binom{n}{i}(a+bx)^{n-i}a^i}{2n-2i+m+2}, \quad n \geq 3, m \geq 3$$

$$I_n = \int \frac{\sqrt{a+bx}}{x^n}\,\mathrm{d}x, \qquad n = 1, 2, 3, \ldots$$

$$I_1 = 2\sqrt{a+bx} + \sqrt{a}\ln\left|\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}\right|, \quad \text{ak } a > 0$$
$$I_1 = 2\sqrt{a+bx} + \frac{2a}{\sqrt{-a}}\operatorname{arctg}\frac{\sqrt{a+bx}}{\sqrt{-a}}, \quad \text{ak } a < 0$$
$$I_n = -\frac{\sqrt{(a+bx)^3}}{(n-1)ax^{n-1}} + \frac{(5-2n)b}{2(n-1)a}I_{n-1}, \quad n \geq 2$$

$$I_n = \int \frac{x^n}{\sqrt{a^2+b^2x^2}}\,\mathrm{d}x, \qquad n = 0, 1, 2, 3, 4, \quad a > 0,\ b > 0$$

$$I_0 = \frac{1}{b}\ln\left|bx + \sqrt{a^2+b^2x^2}\right|$$
$$I_1 = \frac{1}{b^2}\sqrt{a^2+b^2x^2}$$
$$I_2 = \frac{x\sqrt{a^2+b^2x^2}}{2b^2} - \frac{a^2}{2b^3}\ln\left|bx + \sqrt{a^2+b^2x^2}\right|$$
$$I_3 = \frac{\sqrt{(a^2+b^2x^2)^3}}{3b^4} - \frac{a^2}{b^4}\sqrt{a^2+b^2x^2}$$
$$I_4 = \frac{x^3\sqrt{a^2+b^2x^2}}{4b^2} - \frac{3a^2x}{8b^4}\sqrt{a^2+b^2x^2} + \frac{3a^4}{8b^5}\ln\left|bx + \sqrt{a^2+b^2x^2}\right|$$

$$I_n = \int \frac{1}{x^n\sqrt{a^2+b^2x^2}}\,\mathrm{d}x, \qquad n = 1, 2, 3, 4, 5, \quad a > 0,\ b > 0$$

$$I_1 = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2+b^2x^2}}{bx}\right|$$
$$I_2 = -\frac{\sqrt{a^2+b^2x^2}}{a^2x}$$
$$I_3 = -\frac{\sqrt{a^2+b^2x^2}}{2a^2x} + \frac{b^2}{2a^3}\ln\left|\frac{a+\sqrt{a^2+b^2x^2}}{bx}\right|$$
$$I_4 = \frac{2b^2x^2-a^2}{a^4x}\sqrt{a^2+b^2x^2}$$
$$I_5 = \frac{3b^2x^2-2a^2}{8a^4x^4}\sqrt{a^2+b^2x^2} - \frac{3b^4}{8a^5}\ln\left|\frac{a+\sqrt{a^2+b^2x^2}}{bx}\right|$$

$$I_n = \int x^n\sqrt{a^2+b^2x^2}\,\mathrm{d}x, \qquad n = 0, 1, 2, 3\ldots, \quad a > 0,\ b > 0$$

$I_0 = \frac{x\sqrt{a^2+b^2x^2}}{2} + \frac{a^2}{2b} \ln \left| bx + \sqrt{a^2+b^2x^2} \right|$

$I_1 = \frac{\sqrt{(a^2+b^2x^2)^3}}{3b^2}$

$I_2 = \frac{x(a^2+2b^2x^2)}{8b^2}\sqrt{a^2+b^2x^2} - \frac{a^4}{8b^3} \ln \left| bx + \sqrt{a^2+b^2x^2} \right|$

$I_3 = \frac{\sqrt{(a^2+b^2x^2)^5}}{5b^4} - \frac{a^2\sqrt{(a^2+b^2x^2)^3}}{3b^4}$

$I_4 = \frac{x^3\sqrt{(a^2+b^2x^2)^3}}{6b^2} - \frac{a^2x\sqrt{(a^2+b^2x^2)^3}}{8b^4} + \frac{a^4x\sqrt{a^2+b^2x^2}}{16b^4} + \frac{a^6}{16b^5} \ln \left| bx + \sqrt{a^2+b^2x^2} \right|$

$I_n = \frac{x^{n-1}\sqrt{(a^2+b^2x^2)^3}}{(n+2)b^2} - \frac{(n-1)a^2}{(n+2)b^2} I_{n-2}$

$$I_n = \int \frac{\sqrt{a^2+b^2x^2}}{x^n}\,\mathrm{d}x, \qquad n = 1, 2, 3, 4, \quad a > 0,\ b > 0$$

$I_1 = \sqrt{a^2+b^2x^2} - a \ln \left| \frac{a+\sqrt{a^2+b^2x^2}}{bx} \right|$

$I_2 = -\frac{\sqrt{a^2+b^2x^2}}{x} + b \ln \left| bx + \sqrt{a^2+b^2x^2} \right|$

$I_3 = -\frac{\sqrt{a^2+b^2x^2}}{2x^2} - \frac{b^2}{2a} \ln \left| \frac{a+\sqrt{a^2+b^2x^2}}{bx} \right|$

$I_4 = -\frac{\sqrt{(a^2+b^2x^2)^3}}{3a^2b^2x^3}$

$$I_n = \int \frac{x^n}{\sqrt{a^2-b^2x^2}}\,\mathrm{d}x, \qquad n = 0, 1, 2, 3, 4, \quad a > 0,\ b > 0$$

$I_0 = \frac{1}{b} \arcsin \frac{bx}{a}$

$I_1 = -\frac{\sqrt{a^2-b^2x^2}}{b^2}$

$I_2 = -\frac{x\sqrt{a^2-b^2x^2}}{2b^2} + \frac{a^2}{2b^3} \arcsin \frac{bx}{a}$

$I_3 = -\frac{2a^2+b^2x^2}{3b^4}\sqrt{a^2-b^2x^2}$

$I_4 = -\frac{x^2\sqrt{a^2-b^2x^2}}{4b^2} - \frac{3a^2}{8b^4} x\sqrt{a^2-b^2x^2} + \frac{3a^4}{8b^5} \arcsin \frac{bx}{a}$

$$I_n = \int \frac{1}{x^n\sqrt{a^2-b^2x^2}}\,\mathrm{d}x, \qquad n = 1, 2, 3, 4, 5, \quad a > 0,\ b > 0$$

$I_1 = -\frac{1}{a} \ln \left| \frac{a+\sqrt{a^2-b^2x^2}}{bx} \right|$

$I_2 = -\frac{\sqrt{a^2-b^2x^2}}{a^2x}$

$I_3 = -\frac{\sqrt{a^2-b^2x^2}}{2a^2x^2} - \frac{b^2}{2a^3} \ln \left| \frac{a+\sqrt{a^2-b^2x^2}}{bx} \right|$

$I_4 = -\frac{a^2+2b^2x^2}{3a^4x^3}\sqrt{a^2-b^2x^2}$

$I_5 = -\frac{\sqrt{a^2-b^2x^2}}{4a^2x^4} - \frac{3b^2}{8} \frac{\sqrt{a^2-b^2x^2}}{a^4x^2} - \frac{3b^4}{8a^5} \ln \left| \frac{a+\sqrt{a^2-b^2x^2}}{bx} \right|$

$$I_n = \int x^n\sqrt{a^2-b^2x^2}\,\mathrm{d}x, \qquad n = 0, 1, 2, 3 \ldots, \quad a > 0,\ b > 0$$

$I_0 = \frac{x\sqrt{a^2-b^2x^2}}{2} + \frac{a^2}{2b}\arcsin\frac{bx}{a}$

$I_1 = -\frac{\sqrt{(a^2-b^2x^2)^3}}{3b^2}$

$I_2 = \frac{2b^2x^3-a^2x}{8b^2}\sqrt{a^2-b^2x^2} + \frac{a^4}{8b^3}\arcsin\frac{bx}{a}$

$I_3 = \frac{\sqrt{(a^2-b^2x^2)^5}}{5b^4} - \frac{a^2\sqrt{(a^2-b^2x^2)^3}}{3b^4}$

$I_4 = -\frac{x^3\sqrt{(a^2-b^2x^2)^3}}{6b^2} - \frac{a^2x\sqrt{(a^2-b^2x^2)^3}}{8b^4} + \frac{a^4x\sqrt{a^2-b^2x^2}}{16b^4} + \frac{a^6}{16b^5}\arcsin\frac{bx}{a}$

$I_n = -\frac{x^{n-1}\sqrt{(a^2-b^2x^2)^3}}{(n+2)b^2} + \frac{(n-1)a^2}{(n+2)b^2}I_{n-2}, \quad n \geq 2$

$$I_n = \int \frac{\sqrt{a^2-b^2x^2}}{x^n}\,\mathrm{d}x, \qquad n = 1,2,3,4, \quad a>0,\ b>0$$

$I_1 = \sqrt{a^2-b^2x^2} - a\ln\left|\frac{a+\sqrt{a^2-b^2x^2}}{bx}\right|$

$I_2 = -\frac{\sqrt{a^2-b^2x^2}}{x} - b\arcsin\frac{x}{a}$

$I_3 = -\frac{\sqrt{a^2-b^2x^2}}{2x^2} + \frac{b^2}{2a}\ln\left|\frac{a+\sqrt{a^2-b^2x^2}}{bx}\right|$

$I_4 = -\frac{\sqrt{(a^2-b^2x^2)^3}}{3a^2x^3}$

$$I_n = \int \frac{x^n}{\sqrt{b^2x^2-a^2}}\,\mathrm{d}x, \qquad n = -2,-1,0,1,2,3, \quad a>0,\ b>0$$

$I_{-2} = \frac{\sqrt{b^2x^2-a^2}}{a^2x}$

$I_{-1} = \frac{1}{a}\arccos\left|\frac{a}{bx}\right|$

$I_0 = \frac{1}{b}\ln\left|bx+\sqrt{b^2x^2-a^2}\right|$

$I_1 = \frac{\sqrt{b^2x^2-a^2}}{b^2}$

$I_2 = \frac{x\sqrt{b^2x^2-a^2}}{2b^2} + \frac{a^2}{2b^3}\ln\left|bx+\sqrt{b^2x^2-a^2}\right|$

$I_3 = \frac{x^2\sqrt{b^2x^2-a^2}}{b^2} - \frac{2}{3b^4}\sqrt{(b^2x^2-a^2)^3}$

$$I_n = \int x^n\sqrt{b^2x^2-a^2}\,\mathrm{d}x, \qquad n = \ldots,-2,-1,0,1,2,3,\ldots, \quad a>0,\ b>0$$

$I_{-2} = -\frac{\sqrt{b^2x^2-a^2}}{x} + b\ln\left|bx+\sqrt{b^2x^2-a^2}\right|$

$I_{-1} = \sqrt{b^2x^2-a^2} - a\arccos\left|\frac{a}{bx}\right|$

$I_0 = \frac{x\sqrt{b^2x^2-a^2}}{2} - \frac{a^2}{2b}\ln\left|bx+\sqrt{b^2x^2-a^2}\right|$

$I_1 = \frac{1}{3b^2}\sqrt{(b^2x^2-a^2)^3}$

$I_2 = \frac{x\sqrt{(b^2x^2-a^2)^3}}{4b^2} + \frac{a^2}{4b}I_0$

$I_n = \frac{x^{n-1}\sqrt{(b^2x^2-a^2)^3}}{(n+2)b^2} + \frac{(n-1)a^2}{(n+2)b}I_{n-2}$

I.4 Neurčité integrály goniometrických funkcií

$$I_n = \int \sin^n x \, \mathrm{d}x, \qquad n = 1, 2, 3 \dots$$

$I_1 = -\cos x$
$I_2 = \frac{x}{2} - \frac{\sin 2x}{4}$
$I_3 = \frac{\cos^3 x}{3} - \cos x$
$I_n = \frac{\sin^{n-1} x \cos x}{-n} + \frac{n-1}{n} I_{n-2}, \quad n \geq 3$

$$I_n = \int \frac{1}{\sin^n x} \, \mathrm{d}x, \qquad n = 1, 2, 3 \dots$$

$I_1 = \ln \left| \operatorname{tg} \frac{x}{2} \right| = -\frac{1}{2} \ln \left| \frac{1+\cos x}{1-\cos x} \right|$
$I_2 = -\operatorname{cotg} x$
$I_3 = -\frac{\cos x}{2 \sin^2 x} + \frac{1}{2} \left| \operatorname{tg} \frac{x}{2} \right|$
$I_n = \frac{\cos x}{(1-n) \sin^{n-1} x} + \frac{n-2}{n-1} I_{n-2}, \quad n \geq 3$

$$I_n = \int \cos^n x \, \mathrm{d}x, \qquad n = 1, 2, 3 \dots$$

$I_1 = \sin x$
$I_2 = \frac{x}{2} + \frac{\sin 2x}{4}$
$I_3 = \sin x - \frac{\sin^3 x}{3}$
$I_n = \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}, \quad n \geq 3$

$$I_n = \int \frac{1}{\cos^n x} \, \mathrm{d}x, \qquad n = 1, 2, 3 \dots$$

$I_1 = \ln \left| \operatorname{tg} \left(\frac{\pi}{4} + \frac{x}{2} \right) \right|$
$I_2 = \operatorname{tg} x$
$I_3 = \frac{\sin x}{2 \cos^2 x} + \frac{1}{2} \ln \left| \operatorname{tg} \left(\frac{\pi}{4} + \frac{x}{2} \right) \right|$
$I_n = \frac{\sin x}{(n-1) \cos^{n-1} x} + \frac{n-2}{n-1} I_{n-2}, \quad n \geq 3$

$$I_n = \int x^n \sin x \, \mathrm{d}x, \qquad n = 1, 2, 3$$

$I_1 = \sin x - x \cos x$
$I_2 = 2x \sin x + (2 - x^2) \cos x$
$I_3 = (3x^2 - 6) \sin x + (6x - x^2) \cos x$
$I_n = -x^n \cos x + n x^{n-1} \sin x - n(n-1) I_{n-2}, \quad n \geq 3$

$$I_n = \int x^n \cos x \, \mathrm{d}x, \qquad n = 1, 2, 3 \ldots$$

$I_1 = \cos x + x \sin x$
$I_2 = 2x \cos x + (x^2 - 2) \sin x$
$I_3 = (3x^2 - 6) \cos x + (x^3 - 6x) \sin x$
$I_n = x^n \sin x + nx^{n-1} \cos x - n(n-1) I_{n-2}, \quad n \geq 3$

$$I_{n,m} = \int \sin^n x \cos^m x \, \mathrm{d}x, \qquad n = 1, 2, 3 \ldots, \quad m = 1, 2, 3, \ldots$$

$I_{1,1} = \frac{\sin^2 x}{2}$
$I_{1,2} = -\frac{\cos^3 x}{3}$
$I_{1,m} = -\frac{\cos^{m+1} x}{m+1}$
$I_{2,1} = \frac{\sin^3 x}{3}$
$I_{2,2} = \frac{x}{8} - \frac{\sin 4x}{32}$
$I_{2,3} = \frac{\sin^3 x \cos^2 x}{5} + \frac{2}{15} \sin^3 x$
$I_{3,2} = \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3}$
$I_{4,4} = \frac{1}{128} \left(\frac{\sin 8x}{8} - \sin 4x + 3x \right)$

$I_{n,m} = -\frac{1}{m+n} \sin^{n-1} x \cos^{m+1} x + \frac{n-1}{m+n} I_{n-2,m}, \quad n \geq 3$
$I_{n,m} = \frac{1}{m+n} \sin^{n+1} x \cos^{m-1} x + \frac{m-1}{m+n} I_{n,m-2}, \quad m \geq 3$

$$I_{n,m} = \int \frac{1}{\sin^n x \cos^m x} \, \mathrm{d}x, \qquad n = 1, 2, 3 \ldots, \quad m = 1, 2, 3, \ldots$$

$I_{1,1} = \ln |\mathrm{tg}\, x|$
$I_{1,m} = \frac{1}{(m-1) \cos^{m-1} x} + I_{1,m-2}, \quad m \geq 2$
$I_{n,1} = -\frac{1}{(n-1) \sin^{n-1} x} + I_{n-2,1} \quad n \geq 2$
$I_{n,m} = \frac{1}{(m-1) \sin^{n-1} x \cos^{m-1} x} + \frac{n+m-2}{m-1} I_{n,m-2}, \quad m \geq 2$
$I_{n,m} = -\frac{1}{(n-1) \sin^{n-1} x \cos^{m-1} x} + \frac{n+m-2}{m-1} I_{n-2,m}, \quad n \geq 2$

$$I_{n,m} = \int \frac{\sin^n x}{\cos^m x} \, \mathrm{d}x, \qquad n = 1, 2, 3 \ldots, \quad m = 1, 2, 3, \ldots$$

$I_{1,1} = -\ln |\cos x|$
$I_{1,2} = \frac{1}{\cos x}$
$I_{1,3} = \frac{1}{2 \cos^2 x}$
$I_{1,m} = \frac{1}{(m-1) \cos^{m-1} x}, \quad m \geq 2$
$I_{2,1} = -\sin x + \ln \left| \mathrm{tg} \left(\frac{\pi}{4} + \frac{x}{2} \right) \right|$
$I_{2,2} = \mathrm{tg}\, x - x$
$I_{2,3} = \frac{\sin x}{2 \cos^2 x} - \frac{1}{2} \ln \left| \mathrm{tg} \left(\frac{\pi}{4} + \frac{x}{2} \right) \right|$
$I_{2,4} = \frac{\mathrm{tg}^3 x}{3}$
$I_{2,m} = \frac{\sin x}{(m-1) \cos^{m-1} x} - \frac{1}{m-1} \int \frac{1}{\cos^{m-2} x} \, \mathrm{d}x,$
$m \geq 3$
$I_{3,1} = -\frac{\sin^2 x}{2} - \ln |\cos x|$
$I_{3,2} = \cos x + \frac{1}{\cos x}$
$I_{3,3} = \frac{\mathrm{tg}^2 x}{2} + \ln |\cos x|$
$I_{3,4} = \frac{1}{3 \cos^3 x} - \frac{1}{\cos x}$

$I_{n,m} = \frac{\sin^{n+1} x}{(m-1) \cos^{m-1} x} - \frac{n-m+2}{m-1} I_{n,m-2}, \quad m \geq 3$

$I_{n,m} = -\frac{\sin^{n-1} x}{(n-m)\cos^{m-1} x} + \frac{n-1}{n-m} I_{n-2,m}, \quad n \geq 2, \quad n \neq m$

$I_{n,m} = \frac{\sin^{n-1} x}{(m-1)\cos^{m-1} x} - \frac{n-1}{m-1} I_{n-2,m-2}, \quad m \geq 3, \quad n \geq 2$

$$I_{n,m} = \int \frac{\cos^n x}{\sin^m x}\,\mathrm{d}x, \qquad n = 1, 2, 3\ldots, \quad m = 1, 2, 3, \ldots$$

$I_{1,1} = \ln|\sin x|$

$I_{1,2} = -\frac{1}{\sin x}$

$I_{1,3} = -\frac{1}{2\sin^2 x}$

$I_{1,m} = -\frac{1}{(m-1)\sin^{m-1} x}, \quad m \geq 2$

$I_{2,1} = \cos x + \ln\left|\mathrm{tg}\,\frac{x}{2}\right|$

$I_{2,2} = -\mathrm{cotg}\,x - x$

$I_{2,3} = -\frac{\cos x}{2\sin^2 x} + \frac{1}{2}\ln\left|\mathrm{tg}\,\frac{x}{2}\right|$

$I_{2,4} = -\frac{\mathrm{cotg}^3 x}{3}$

$I_{2,m} = -\frac{\cos x}{(m-1)\sin^{m-1} x} - \frac{1}{m-1}\int \frac{1}{\sin^{m-2} x}\,\mathrm{d}x,$
$m \geq 2$

$I_{3,1} = \frac{\cos^2 x}{2} + \ln|\sin x|$

$I_{3,2} = -\sin x - \frac{1}{\sin x}$

$I_{3,3} = -\frac{\mathrm{cotg}^2 x}{2} - \ln|\sin x|$

$I_{3,4} = \frac{1}{\sin x} - \frac{1}{3\sin^3 x}$

$I_{n,m} = -\frac{\cos^{n+1} x}{(m-1)\sin^{m-1} x} - \frac{n-m+2}{m-1} I_{n,m-2}, \quad m \geq 3$

$I_{n,m} = \frac{\cos^{n-1} x}{(n-m)\sin^{m-1} x} + \frac{n-1}{n-m} I_{n-2,m}, \quad n \geq 2, \quad n \neq m$

$I_{n,m} = -\frac{\cos^{n-1} x}{(m-1)\sin^{m-1} x} - \frac{n-1}{m-1} I_{n-2,m-2}, \quad m \geq 3, \quad n \geq 2$

I.5 Doplnok

$\int x^n\, e^{ax}\,\mathrm{d}x = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}\, e^{ax}\,\mathrm{d}x,\ a \neq 0$

$\int e^{ax}\ \sin\ bx\,\mathrm{d}x = \frac{e^{ax}\ (a\ \sin\ bx - b\ \cos\ bx)}{a^2+b^2}$

$\int e^{ax}\ \cos\ bx\,\mathrm{d}x = \frac{e^{ax}\ (a\ \cos\ bx + b\ \sin\ bx)}{a^2+b^2}$