1. Nájdite riešenie danej diferenciálnej rovnice.

a) $u_{xx} = 3x^2y + x\cos y$

b) $u_{xy} = y^2\cos 2x + 3\mathrm{e}^{2x}y$

c) $u_{yy} = \frac{x}{y^2+1} \qquad u(x,0) = x^2, u_y(x,0) = \sin x$

d) $u_{yx} = x\cos(3x+2y)$

2. Nájdite riešenie danej diferenciálnej rovnice.

a) $u_{xx} - 4yu_x + 3y^2u = 0$

b) $u_{yx} + \sin x u_y = \sin x$

c) $xu_{yx} + u_y = x\cos x$

d) $yu_{xy} + u_x = x\mathrm{e}^y$

e) $u_{yy} - x^2u_y - 6x^4u = 0$

f) $u_{yy} - 4xu_y + 4x^2u = 0;$
$u(x,0) = x^2, u(x,1) = \mathrm{e}^{2x}$

g) $u_{yy} - 5xu_y + 4x^2u = 7x$
$u(x,1) = \frac{7}{4x}, u(x,-1) = 0$

h) $u_{xx} - 3yu_x - 4y^2u = \ln y$

i) $\sin y\ u_{xx} - 2\cos y\ u_x - \sin y\ u = \mathrm{tg}\, y$

Výsledky:

1a. $u(x,y) = \frac{x^4}{4}y + \frac{x^3}{6}\cos y + xf(y) + g(y),$

1b. $u(x,y) = \frac{y^3}{6}\sin 2x + \frac{3}{4}\mathrm{e}^{2x}y^2 + H(x) + g(y),$

1c. $u(x,y) = xy\ \mathrm{arctg}\ y - \frac{x}{2}\ln(y^2+1) + y\sin x + x^2,$

1d. $u(x,y) = -\frac{x}{6}\cos(3x+2y) + \frac{1}{18}\sin(3x+2y) + F(y) + g(x),$

2a. $u(x,y) = f(y)\mathrm{e}^{3xy} + g(y)\mathrm{e}^x y,$

2b. $u(x,y) = G(y)\mathrm{e}^{\cos x} + y + f(x),$

2c. $u(x,y) = y\sin x + y\frac{\cos x}{x} + \frac{1}{x}F(y) + g(x),$

2d. $u(x,y) = \frac{x^2}{2y}\mathrm{e}^y + \frac{1}{y}F(x) + g(y),$

2e. $u(x,y) = f(x)\mathrm{e}^{-2x^2y} + g(x)\mathrm{e}^{3x^2y},$

2f. $u(x,y) = x^2\mathrm{e}^{2xy} + (x^2-1)y\mathrm{e}^{2xy},$

2g. $u(x,y) = -\frac{7}{4x}\left(\frac{\mathrm{e}^{4xy}}{\mathrm{e}^{-4x}-\mathrm{e}^{2x}} - \frac{\mathrm{e}^{3x+xy}}{\mathrm{e}^{-4x}-\mathrm{e}^{2x}} - 1\right),$

2h. $u(x,y) = f(y)\mathrm{e}^{4xy} + g(y)\mathrm{e}^{-xy} - \frac{\ln y}{4y^2},$

2i. $u(x,y) = f(y)\mathrm{e}^{x\frac{\cos y+1}{\sin y}} + g(y)\mathrm{e}^{x\frac{\cos y-1}{\sin y}} - \frac{1}{\cos y},$