

1. Riešte dané nehomogénne systémy diferenciálnych rovníc.

a)

$$\begin{aligned} y_1' &= 3y_1 - 2y_2 \\ y_2' &= 2y_1 - y_2 + 1 \end{aligned}$$

b)

$$\begin{aligned} y_1' &= y_1 - 2y_2 + 3 \\ y_2' &= y_1 - y_2 + 1 \end{aligned}$$

c)

$$\begin{aligned} y_1' &= 3y_1 + 2y_2 + 7 \\ y_2' &= y_1 + 2y_2 + 5 \end{aligned}$$

d)

$$\begin{aligned} y_1' &= -5y_1 + 2y_2 + e^x \\ y_2' &= 6y_1 + 6y_2 + e^{2x} \end{aligned}$$

e)

$$\begin{aligned} y_1' &= y_1 - y_2 + \frac{1}{\cos x} \\ y_2' &= 2y_1 - y_2 \end{aligned}$$

f)

$$\begin{aligned} y_1' &= -y_1 + y_2 + \cos x \\ y_2' &= -5y_1 + 3y_2 \end{aligned}$$

g)

$$\begin{aligned} y_1' &= 2y_1 + 4y_2 + \cos x \\ y_2' &= -y_1 - 2y_2 + \sin x \end{aligned}$$

h)

$$\begin{aligned} y_1' &= y_2 + 2e^x \\ y_2' &= y_1 + x^2 \end{aligned}$$

i)

$$\begin{aligned} y_1' &= y_2 - 5\sin x \\ y_2' &= -4y_1 + 17\cos x \end{aligned}$$

j)

$$\begin{aligned} y_1' &= 3y_1 + 12y_2 - 4y_3 + 2 \\ y_2' &= -y_1 - 3y_2 + y_3 + 1 \\ y_3' &= -y_1 - 12y_2 + 6y_3 + 1 \end{aligned}$$

k)

$$\begin{aligned} y_1' &= 2y_1 + y_2 - 2y_3 + 2 - x \\ y_2' &= -y_1 + 1 \\ y_3' &= y_1 + y_2 - y_3 + 1 - x \end{aligned}$$

l)

$$\begin{aligned} y_1' &= -y_1 + y_2 + e^x \\ y_2' &= y_1 - y_2 + e^x \\ y_1(0) &= y_2(0) = 1 \end{aligned}$$

m)

$$\begin{aligned} y_1' &= -y_1 - 2y_2 + 2e^{-x} \\ y_2' &= -y_2 - y_3 + 1 \\ y_3' &= -y_3 + 1 \\ y_1(0) &= y_2(0) = y_3(0) = 1 \end{aligned}$$

**Výsledky:**

**1a.**  $y_1 = c_1 e^x + c_2(1 + 2x)e^x - 2$ ,  $y_2 = c_1 e^x + 2c_2 x e^x - 3$

**1b.**  $y_1 = 2c_1 \cos x + 2c_2 \sin x + 1$ ,  $y_2 = (c_1 - c_2) \cos x + (c_1 + c_2) \sin x + 2$

**1c.**  $y_1 = c_1 e^x + 2c_2 e^{4x} - 1$ ,  $y_2 = -c_1 e^x + c_2 x e^{4x} - 2$

**1d.**  $y_1 = c_1 e^{7x} + 2c_2 e^{-6x} - \frac{917}{390} e^x - \frac{3}{260} e^{2x}$ ,  $y_2 = 6c_1 e^{7x} - c_2 x e^{-6x} - \frac{839}{65} e^x - \frac{101}{520} e^{2x}$

**1e.**  $y_1 = c_1 \cos x + c_2 \sin x + x(\cos x + \sin x) + (\cos x - \sin x) \ln |\cos x|$ ,  
 $y_2 = (c_1 - c_2) \cos x + (c_1 + c_2) \sin x + 2 \cos x \ln |\cos x| + 2x \sin x$

**1f.**  $y_1 = c_1 \cos x + c_2 \sin x + 1 + \frac{x}{2}(\cos x - 2 \sin x) + (\cos x - \frac{3}{4} \sin x) \frac{\cos 2x}{4}$ ,  
 $y_2 = (2c_1 + c_2) \cos x + (-c_1 + 2c_2) \sin x + 2 - \frac{5}{2} x \sin x - \cos 2x (\frac{5}{4} \cos x - \frac{7}{4} \sin x)$

**1g.**  $y_1 = 2c_1 + c_2(-1 - 2x) + 8x \cos x - 5 \sin x + 6 \cos x,$

$y_2 = -c_1 + c_2x - 2 \cos x + 2 \sin x - 4x \cos x$

**1h.**  $y_1 = c_1e^x + c_2e^{-x} + xe^x - x^2 - 2, y_2 = c_1e^x - c_2e^{-x} + (x - 1)e^x - 2x$

**1i.**  $y_1 = c_1 \cos 2x + c_2 \sin 2x + 4 \cos x, y_2 = -2c_1 \sin 2x + 2c_2 \cos 2x + \sin x$

**1j.**  $y_1 = -2c_1e^x - 8c_2e^{2x} - 3c_3e^{3x} + 6, y_2 = c_1e^x + 3c_2e^{2x} + c_3e^{3x} + \frac{5}{6},$   
 $y_3 = 2c_1e^x + 7c_2e^{2x} + 3c_3e^{3x} + \frac{15}{2}$

**1k.**  $y_1 = c_1e^x + c_2 \sin x + c_3 \cos x, y_2 = -c_1e^x + c_2 \cos x - c_3 \sin x + x,$   
 $y_3 = c_2 \sin x + c_3 \cos x + 1$

**1l.**  $y_1 = e^x, y_2 = e^x$

**1m.**  $y_1 = e^{-x}, y_2 = e^{-x}, y_3 = 1$